Statistics for data science: what are the essentials?

David Spiegelhalter

Chair of the Winton Centre for Risk & Evidence Communication, University of Cambridge

President, Royal Statistical Society (2017-2018)

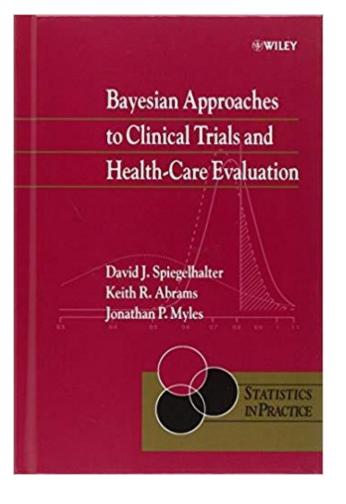
ISM Consortium for Human Resource Development of Statistical Experts in Japan. February 2022 Information Science and Statistics

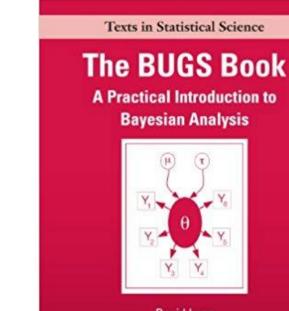
Robert G. Cowell · A. Philip Dawid Steffen L. Lauritzen · David J. Spiegelhalter

Probabilistic Networks and Expert Systems

Exact Computational Methods for Bayesian Networks

🙆 Springer

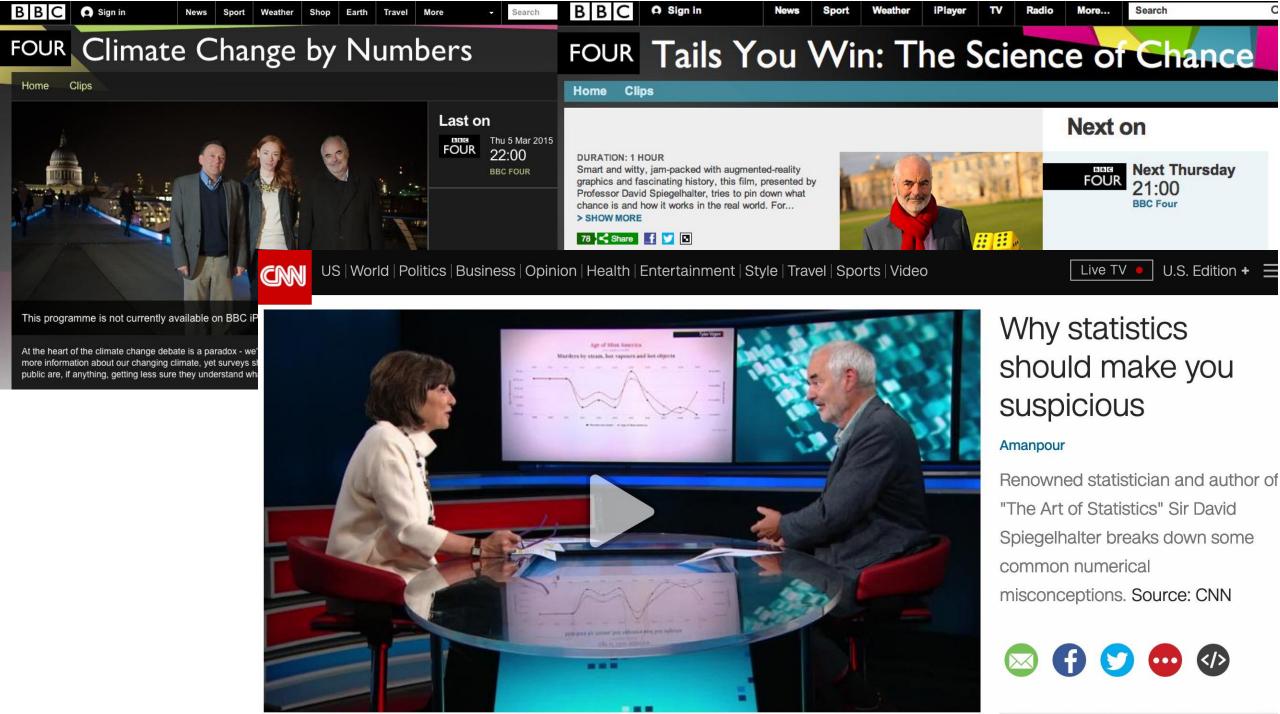




David Lunn Christopher Jackson Nicky Best Andrew Thomas David Spiegelhalter



I used to do statistical methodology.... until I was philanthropically funded in 2007.....

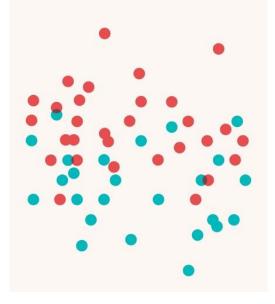


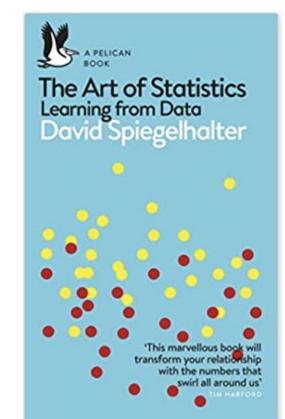


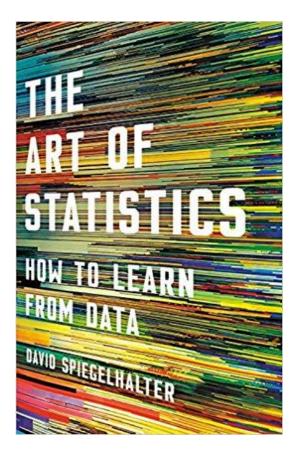


A PELICAN BOOK

The Art of Statistics Learning from Data David Spiegelhalter







Interpreting data is not easy

INTRODUCTION

The numbers have no way of speaking for themselves. We speak for them. We imbue them with meaning.

— Nate Silver, The Signal and the Noise¹

The traditional statistics course

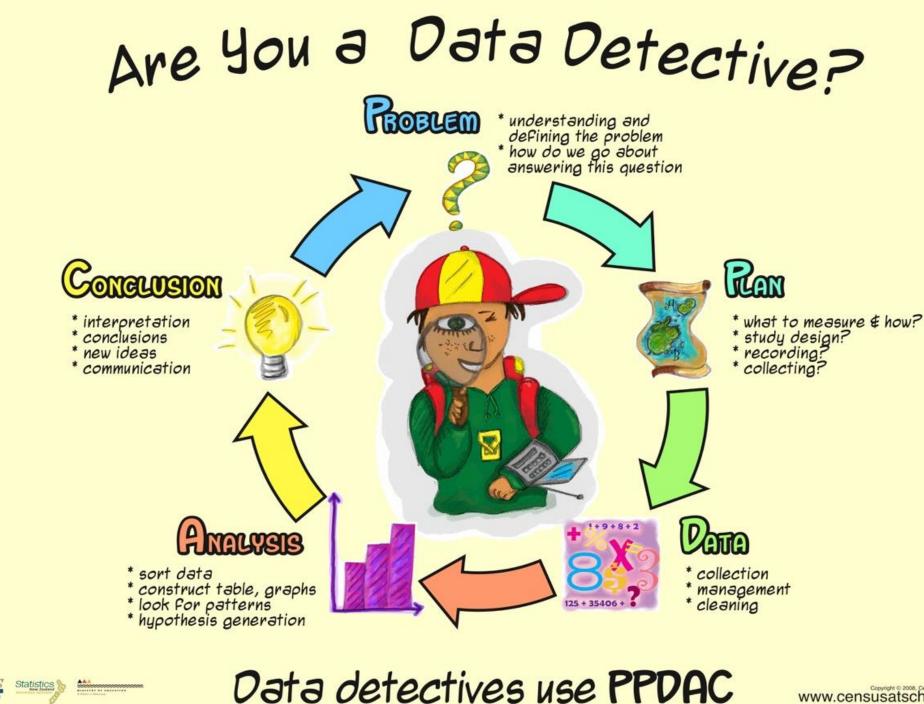
- Describing data with summary statistics o *dull*
- Probability theory for drawing random observation from a population distribution
 - o *difficult and mathematical*
- Probability theory for distributions of summary statistics o mathematical and incomprehensible
- Formulae for statistical tests
 - o mathematical, unmotivated, just a bag of tools
- (If lucky) Examples of using statistical models in real life.

A 'modern' statistical course

- Motivate by problem solving
- Start with visualisation and exploring data
- Focus on what can be reasonably learned from data, biases in data, concluding causation, etc
- Models and algorithms
- Assessing uncertainty through re-sampling data ('bootstrap')
- Probability theory as neat way of turning random variation into uncertainty about what is true
- Hypothesis testing and its potential problems
- Bayesian methods

All these rather abstract, challenging, ideas are there to help answer real questions

- The 'data cycle'
- eg PPDAC (promoted in New Zealand)



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Looking at data

What was the pattern of Harold Shipman's murders?



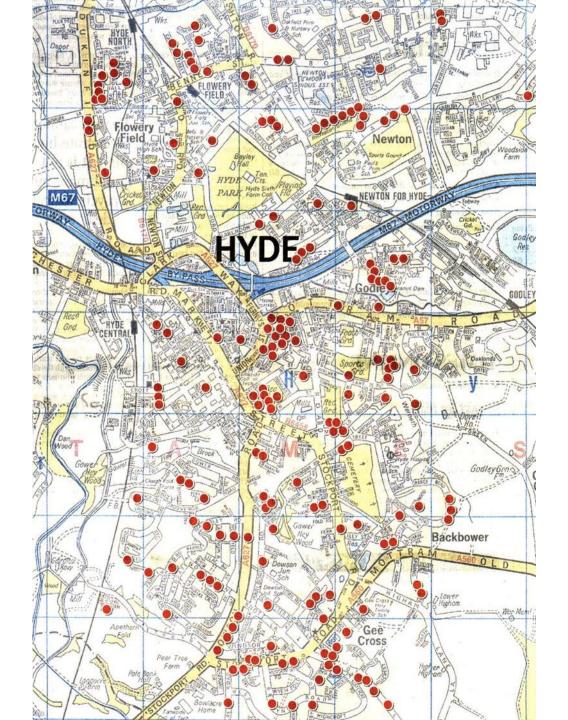
'I have nothing to hide'

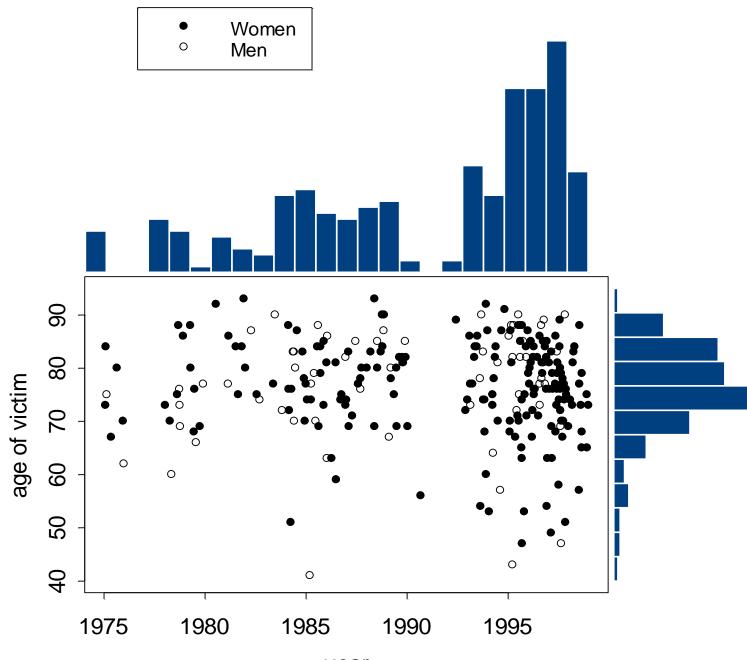
Dr Harold Shipman, general practitioner, on his arrest in September 1998

Shipman Inquiry July 2002:

215 definite victims,

45 probable





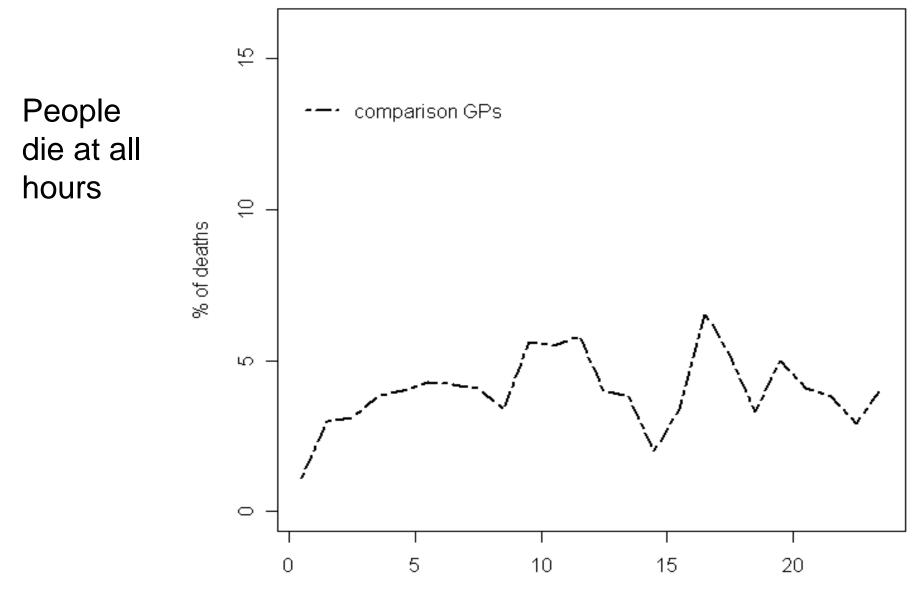
year

Looking at data

What was the pattern of Harold Shipman's murders?

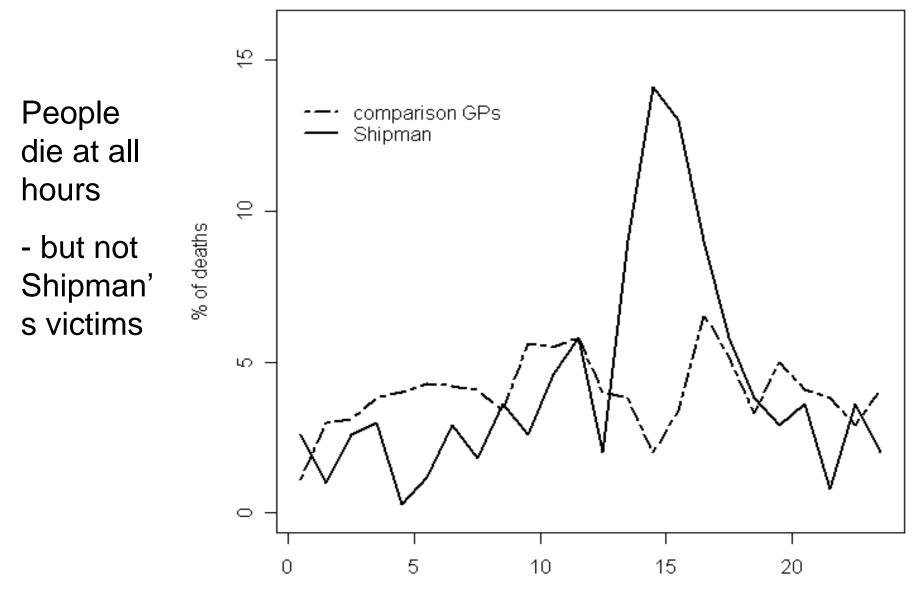
- **Problem**: can more detail tell us more about what Shipman did?
- Plan: compare actual times at which his patients died with the times of deaths recorded by other local GPs
- **Data**: a huge exercise requiring examination of death certificates
- Analysis: simple plotting.....

% of deaths in each hour of the day





% of deaths in each hour of the day



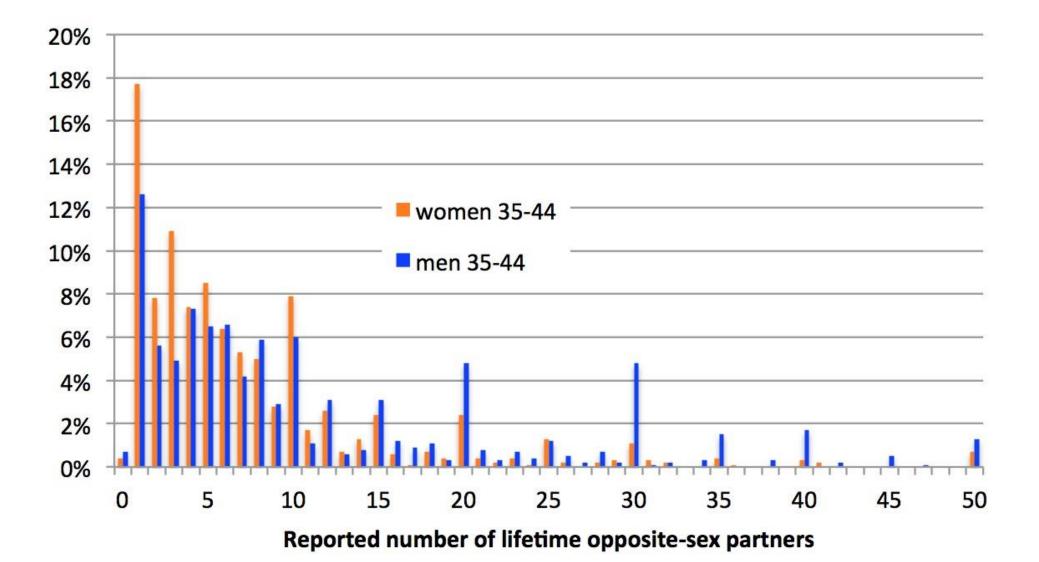
hour

Inference and bias

How many sexual partners have people in Britain had in their lifetime?

- Problem: cannot know this as a fact
- Plan: survey in which people are carefully asked about the sexual activity (Natsal)
- Data: reports of numbers of partners
- Analysis: plotting and summary statistics

How many sexual partners do people report?



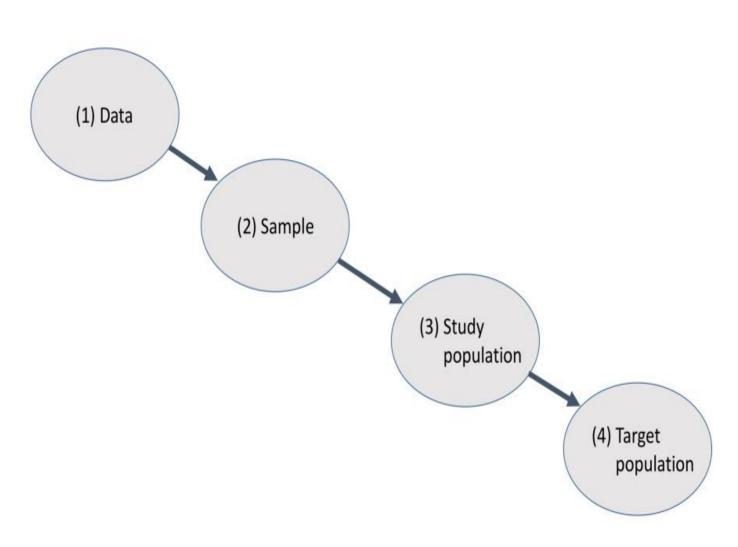
Inference and bias

How many sexual partners have people in Britain **really** had in their lifetime?

Reported number of sexual partners in lifetime	Men aged	Women aged
	35–44	35–44
Mean	14.3	8.5
Median	8	5
Mode	1	1
Range	0 to 500	0 to 550
Inter-quartile range	4 to 18	3 to 10
Standard deviation	24.2	19.7

• Conclusions: can we generalise this to the whole population?????

Induction: the stages in generalising from data



- **1 to 2.** How reliable are the reports?
- Poor memory, social acceptability bias etc
- 2 to 3. How representative is the sample of those eligible for the study?
- Random sampling of families (soup), 66% response
- **3 to 4.** How close does the study population match the target population?
- No people in institutions, etc

Causation (or correlation) The power of the press release....

Socioeconomic position and the risk of brain tumour: a Swedish national population-based cohort study

Amal R Khanolkar,^{1,2} Rickard Ljung,² Mats Talbäck,² Hannah L Brooke,² Sofia Carlsson,² Tiit Mathiesen,³ Maria Feychting²

- abstract:
 - We observed consistent associations between higher socio-economic position and higher risk of glioma
- press release
 - High levels of education linked to heightened brain tumour risk
- Daily Mirror...



🚺 · Science · tumour

Why going to university increases risk of getting a brain tumour

23:30, 20 JUN 2016 BY ANDREW GREGORY

Highly educated people are more likely to suffer from brain tumours than those who do not progress as far in their education



Are Your Saving: Enough to Retire

If you have a £250,00 portfolio, download th

Regression, prediction and algorithms *Who was the luckiest person on the Titanic?*





Ilfracombe, North Devon

ALSO OF FRANCIS WILLIAM SON OF THE ABOVE WHO PERISHED N THE "TITANIC DISASTER APRIL 14 - 1912, ACED 30 YEARS

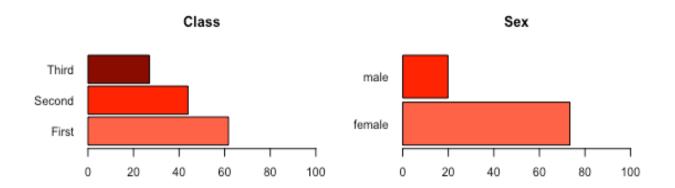


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	3	0	Spector, Mr. Woolf	male		0	0	A.5. 3236	8.0500		S		
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ŀ	3	0	Stankovic, Mr. Ivan	male	33	0	0	349239	8.6625		С		
5	3	1	Stanley, Miss. Amy Zillah Elsie	female	23	0	0	CA. 2314	7.5500		S	С	
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William Somerton's entry in a public database of 1309 passengers (39% survive)

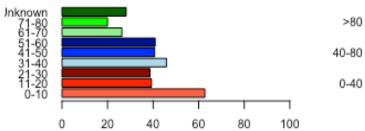
- Challenge: can we build an algorithm that will accurately predict who survives the Titanic?
- Based on factors in data-base, produce either a yes/no judgement, or a probability of survival
- Split the data-base of 1309 passengers at random into a training set (70%) on which to build algorithms, and a test set (30%) to assess how good it is.
- Currently over 59,000 entries in a similar online Kaggle competition

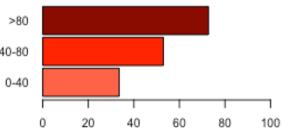
Unsurprising factors predict survival



Age

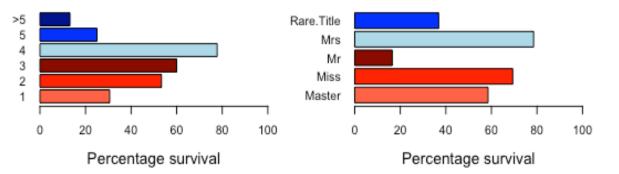




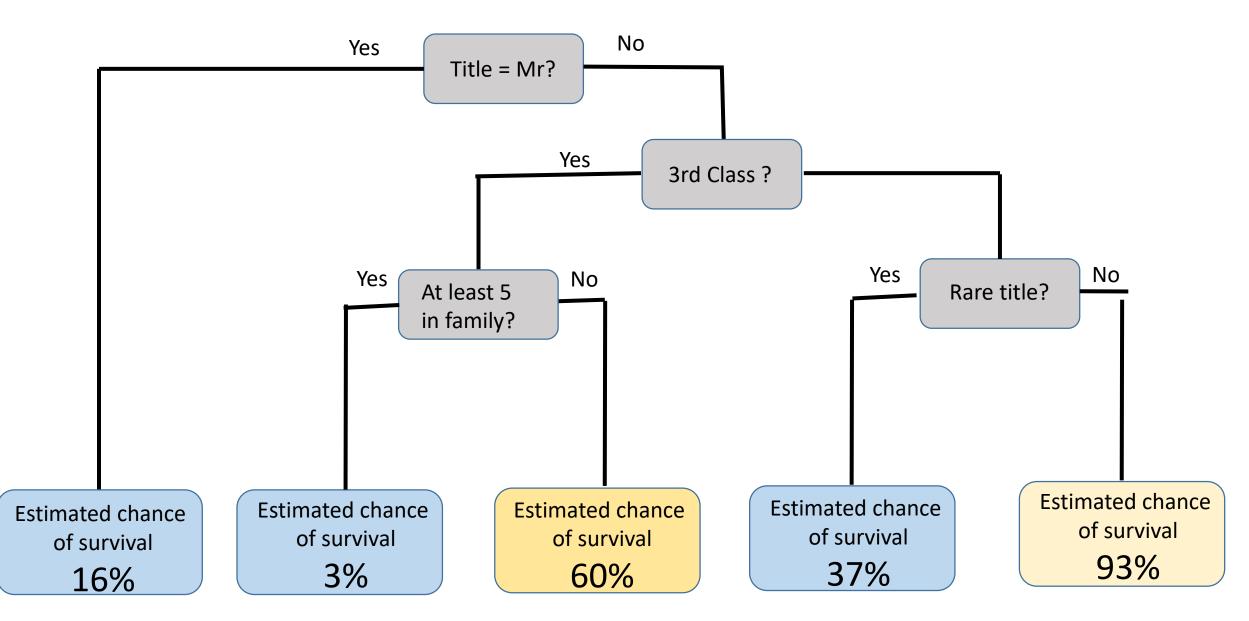


Family size

Title



A simple classification tree



How good is my algorithm?

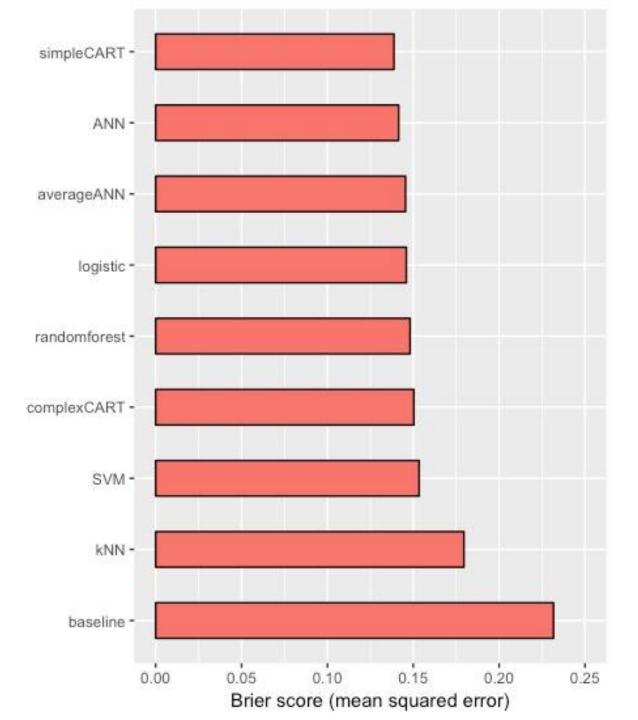
• 'Accuracy' is a very crude way of judging an algorithmic prediction

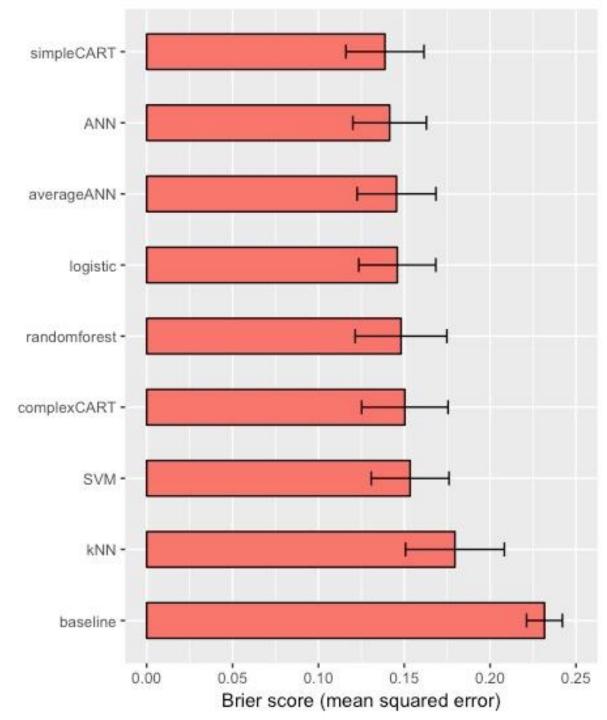
• Better to use the probabilities provided

 If probability p is given to an event X (0,1), then the Brier score is (X-p)²

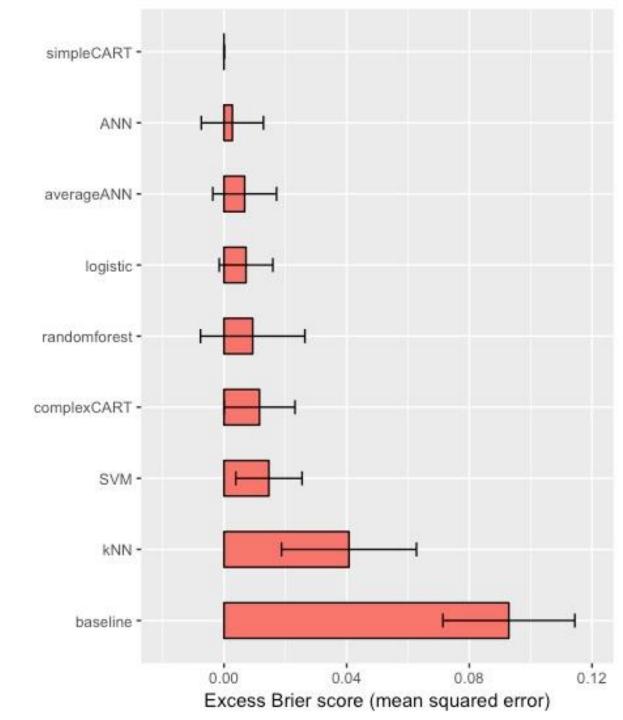
Performance of a range of methods on the test set

Method	Accuracy (high is good)	Brier score (low is good)
Everyone has a 39% chance of surviving	0.639	0.232
All females survive, all males do not	0.786	0.214
Simple classification tree	0.806	0.139
Classification tree (over-fitted)	0.806	0.150
Logistic regression	0.789	0.146
Random forest	0.799	0.148
Support Vector Machine (SVM)	0.782	0.153
Neural network	0.794	0.146
Averaged neural network	0.794	0.142
K-nearest-neighbour	0.774	0.180





- Potentially a very misleading graphic!
- When comparing, need to acknowledge that tested on same cases
- Calculate differences and their standard error
- How confident can we be that simple CART is best algorithm?



Ranking of algorithms

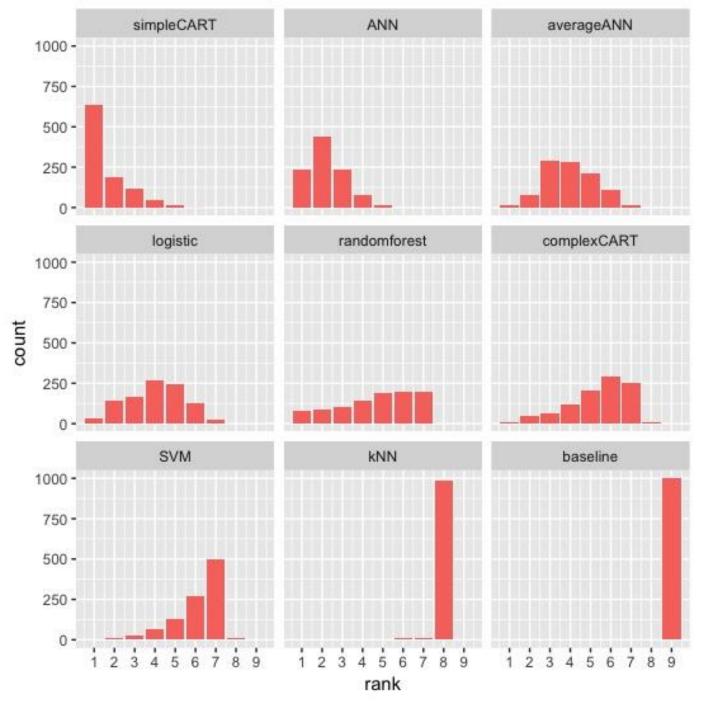
- Bootstrap sample from test set (ie sample of same size, drawn with replacement)
- Rank algorithms by performance on the bootstrap sample
- Repeat '000s of times

 (ranks actual *algorithm* – if want to rank *methods*, need to bootstrap training data too, and reconstruct algorithm each time)

Distribution of true rank of each algorithm

Probability of 'best':

63% simpleCART 23% ANN 8% randomforest



Who was the luckiest person on the Titanic?

- Karl Dahl, a 45-year-old Norwegian/Australian joiner travelling on his own in third class, paid the same fare as Francis Somerton
- Had the lowest average Brier score among survivors – a very surprising survivor
- He apparently dived into the freezing water and clambered into Lifeboat 15, in spite of some on the lifeboat trying to push him back.

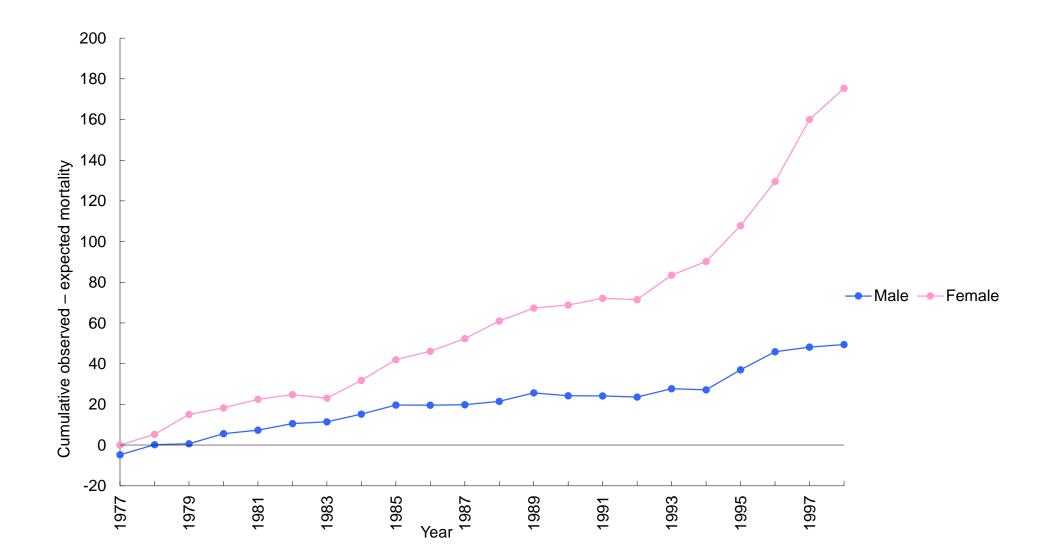
• Hannah Somerton was left just £5, less than Francis spent on his ticket.





Hypothesis testing *Could Harold Shipman have been caught earlier?*

- Using mortality rates from local GPs, calculate how many deaths he would have been **expected** to observe each year, under the **null hypothesis** that his mortality rates were normal.
- Subtract expected from observed number to get excess mortality



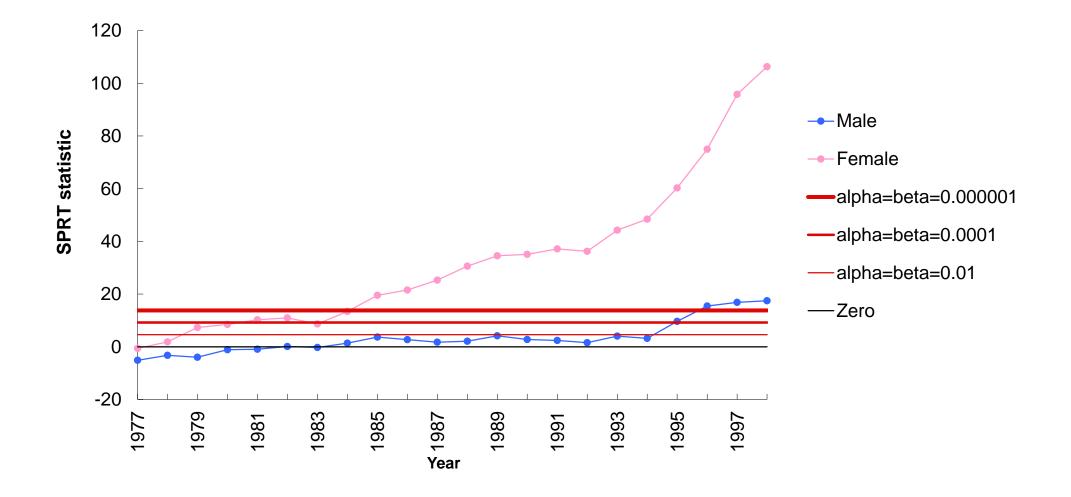
(NB: Shipman Inquiry total of definite or probable victims: **189** female > 65, **55** male over 65)

Hypothesis testing

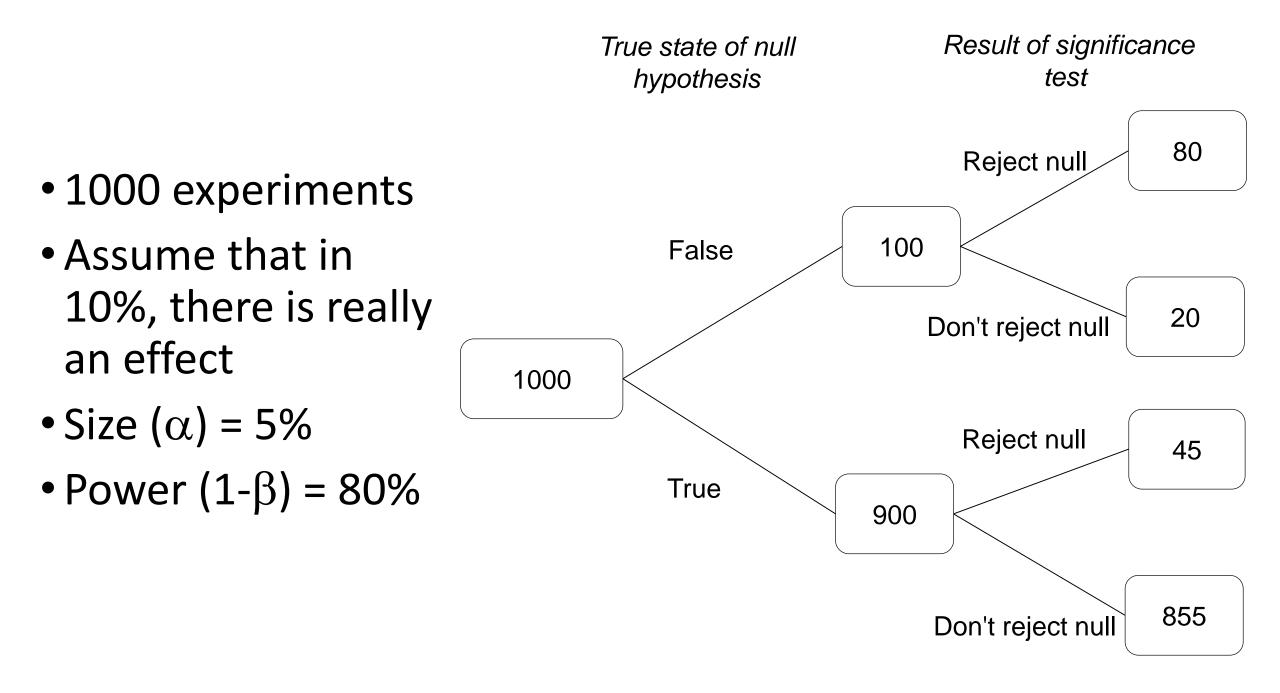
Could Harold Shipman have been caught earlier?

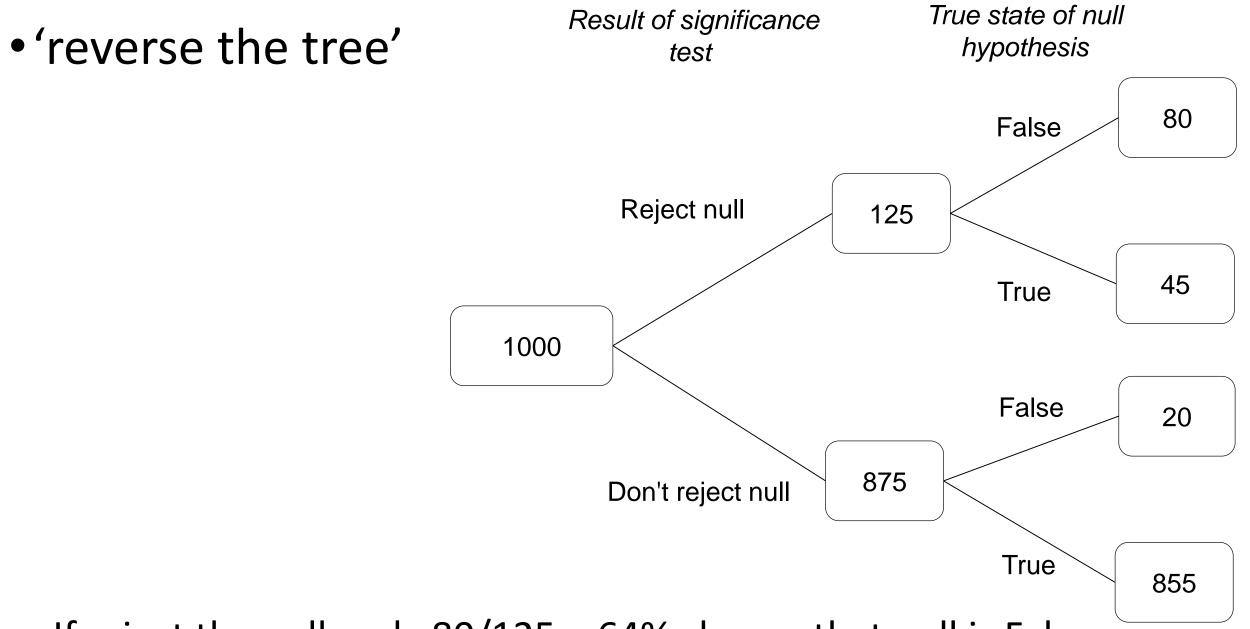
- But when to 'blow the whistle'?
- This are two possible types of error -
 - Type I error: falsely accuse an innocent person
 - Type II error: miss someone with true increased risk
- This is an example of a *hypothesis test* used throughout science. Again, two possible types of error
 - **Type I error:** falsely claim an 'effect' when nothing is there (ie the *null hypothesis* is true)
 - Type II error: miss a true effect
- Generally, we want to
 - control the probability of a Type I error at a low value (α)
 - make experiments large enough to make Type II errors rare (β)

Shipman: "Sequential probability ratio test" (SPRT) older females would have set off 'alarm' in 1985, after only 40 deaths



Probability and Bayes





• If reject the null, only 80/125 = 64% chance that null is False

Bayes theorem

- Initial odds that null hypothesis False = 10 / 90
- After 'significant' results, final odds that null hypothesis False = 80/45
- Likelihood ratio = $\frac{\Pr(\text{significant result} \mid \text{null hypothesis False})}{\Pr(\text{significant result} \mid \text{null hypothesis True})}$

$$=\frac{\text{Power}}{\text{Size}} = \frac{1-\beta}{\alpha} = \frac{0.80}{0.05} = 16$$

• Bayes theorem:

the initial odds for the hypothesis x the likelihood ratio

= the final odds for a hypothesis.

$$\frac{10}{90} \times \frac{80}{5} = \frac{80}{45}$$

Probability and Bayes

What is the probability that the skeleton in a Leicester car park was really Richard III?

A recent case

- On Saturday 25 August 2012, archeologists started digging in a car park in Leicester the site of Grey Friars friary
- In a few hours they found their first skeleton



• This was later claimed to be Richard III



ARTICLE

Received 5 Aug 2014 | Accepted 21 Oct 2014 | Published 2 Dec 2014

DOL 10.1038/ncomms6631

OPEN

Identification of the remains of King Richard III

Turi E. King^{1,2}, Gloria Gonzalez Fortes^{3,4,*}, Patricia Balaresque^{5,*}, Mark G. Thomas⁶, David Balding⁶, Pierpaolo Maisano Delser¹, Rita Neumann¹, Walther Parson^{7,8}, Michael Knapp⁹, Susan Walsh^{10,11}, Laure Tonasso⁵, John Holt¹², Manfred Kayser¹¹, Jo Appleby², Peter Forster^{13,14}, David Ekserdjian¹⁵, Michael Hofreiter^{3,4} & Kevin Schürer¹⁶

probability of evidence, if skeleton were Richard III

Likelihood ratio =

probability of evidence, if someone else

Suggested 'verbal equivalents' for bands of likelihood ratios

Value of likelihood ratio	Verbal equivalent
>1-10	Weak support for proposition
10-100	Moderate support
100-1000	Moderately strong support
1000-10,000	Strong support
10,000-1,000,000	Very strong
>1,000,000	Extremely strong

Standards for the formulation of evaluative forensic science expert opinion

Evidence	Likelihood ratio (conservative estimate)	Verbal equivalent
Radiocarbon dating AD 1456– 1530	2	Weak support
Age and sex of skeleton	5	Weak support
Scoliosis	212	Moderately strong support
Post-mortem wounds	42	Moderate support
mtDNA match	478	Moderately strong support
Y chromosome not matching	0.2	Weak evidence against
Combined evidence	6.5 million	More than extremely strong support

Conclusions – statistics for data science

- Motivate by problem solving
- Start with visualisation and exploring data
- Focus on concepts: what can be reasonably learned from data, biases, causation, etc
- Models and algorithms
- Probability can come much later
- Conditional probability / Bayes theorem taught through 'expected frequency trees/